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Maximum Re-entry Drag Deceleration Revisited

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Abstract

THE maximum re-entry drag deceleration problem is revisited. It is shown that peak deceleration depends on drag coefficient except when an exponential air density profile is assumed. In the sequel, closed-form expressions for peak deceleration are derived for constant drag coefficient. Two simplified air density models are compared. The reader is referred to the complete paper for the analysis using the 1976 Standard Atmosphere density model.

Contents

Flat Earth equations of motion in a uniform gravity field with constant acceleration g are

$$\frac{d}{dh}(u^2) + \frac{k\rho(h)}{\sin\gamma}u^2 = -2g \qquad u^2\frac{d}{dh}(\cos\gamma) - g\cos\gamma = 0 \tag{1}$$

Although a flat Earth is assumed, atmospheric motion due to Earth rotation is modeled as a translation, or a horizontal wind with constant magnitude (465 m/s). The vehicular velocity vector relative to the translating atmosphere has magnitude u and flight-path angle γ, measured positive above the horizontal plane. Altitude h is chosen as the independent variable since air density ρ depends on h. The aerodynamic area-to-mass ratio k is the drag coefficient multiplied by the vehicle area-to-mass ratio. Not included are aerodynamic lift, lift-induced drag, and changes in area and mass due to ablation.

Two simplifing assumptions are introduced to derive a well-known closed-form solution.[1] If gravity is neglected ($g=0$), the re-entry trajectory is rectilinear along the pierce point relative velocity vector. Moreover, if k is constant and equal to its inviscid, hypersonic value k_∞, the solutions are

$$u^2(h) = U^2\exp\left\{-\frac{k_\infty\sigma(h)}{|\sin\Gamma|}\right\}$$

$$\gamma(h) = \Gamma = \text{const} \qquad \sigma(h) = \int_h^H \rho(s)\,ds \tag{2}$$

where (U,Γ) are the values of (u,γ) at the pierce point altitude H. The constant k_∞ has been factored from the density quadrature σ. Recently, closed-form solutions have been obtained for certain integrable, velocity-dependent drag coefficient models.[2] Two cases that should be treated separately are re-entry from circular orbit ($\Gamma=0$ deg) and vertical descent ($\Gamma=-90$ deg). The latter case has a different closed-form solution including gravity.

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For a rectilinear trajectory and constant k_∞, drag deceleration Q is a function of h only:

$$Q = \tfrac{1}{2}k_\infty\rho u^2 = \tfrac{1}{2}U^2k_\infty\rho(h)\exp\left\{-\frac{k_\infty\sigma(h)}{|\sin\Gamma|}\right\} \tag{3}$$

Peak deceleration occurs when $dQ/dh=0$, which is fulfilled if

$$k_\infty\rho = -|\sin\Gamma|\frac{1}{\rho}\frac{d\rho}{dh} \tag{4}$$

If Eq. (4) were solved explicitly for the maximizing altitude h^*, h^* would be a function of k_∞, Γ, and the parameters of the air density model. Substituting Eq. (4) in Eq. (3), it follows that peak deceleration Q^* is

$$Q^* = -\tfrac{1}{2}U^2|\sin\Gamma|\left[\frac{1}{\rho}\frac{d\rho}{dh}\exp\left\{-\frac{k_\infty\sigma}{|\sin\Gamma|}\right\}\right] \tag{5}$$

where it is understood that all altitude-dependent functions in brackets are evaluated at h^*.

Thus far in the development, no assumptions have been made concerning the air density model. For an atmosphere in hydrostatic equilibrium and obeying the ideal gas law, it is well known that ρ is specified by the temperature profile, as follows.

In an isothermal atmosphere, temperature is constant throughout. It can then be shown that[3]

$$\rho(h) = \rho(h_0)\exp\left\{-\frac{h-h_0}{B}\right\} \qquad \frac{1}{\rho}\frac{d\rho}{dh} = -\frac{1}{B} = \text{const}$$

$$\sigma(h) = B[\rho(h) - \rho(H)] \cong B\rho(h) \tag{6}$$

where h_0 is the base altitude and B is the "scale height." The last result assumes that $H \gg B$. Substituting Eq. (6) in Eq. (5), it follows that

$$Q^* = \frac{U^2|\sin\Gamma|}{2Be} \tag{7}$$

which is the Allen and Eggers[1] result; namely, Q^* is independent of k_∞.

In the adiabatic atmosphere, temperature decreases (or increases) with increasing altitude if the atmosphere is heated from below (or above). If temperature decreases linearly with altitude, it can be shown that[3]

$$\rho(h) = \rho(h_0)\left[1-\left(\frac{h-h_0}{B}\right)\right]^b \qquad \frac{1}{\rho}\frac{d\rho}{dh} = \frac{-b}{B-(h-h_0)}$$

$$\sigma(h) \cong \frac{\rho(h)}{1+b}[B-(h-h_0)] \tag{8}$$

The positive constants b and B involve the magnitude of the temperature gradient. For simplicity it was assumed that $h-h_0 \le B$ and $\rho(H) \approx 0$. Substituting Eq. (8) in Eq. (5), it

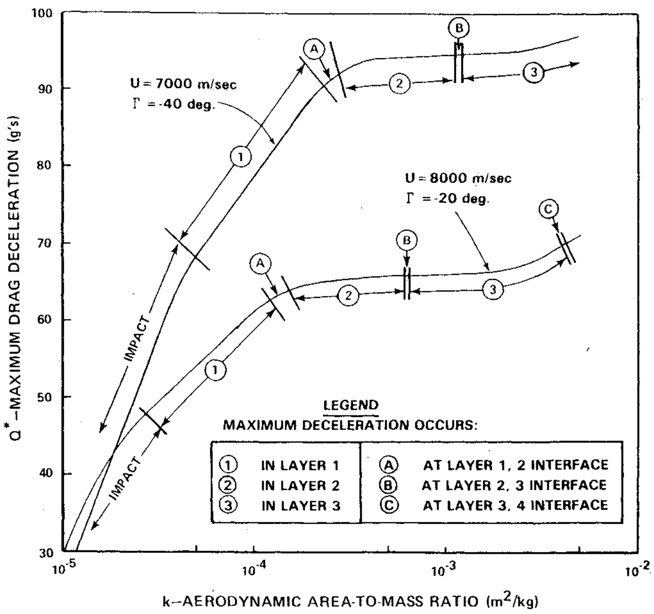

Fig. 1 Maximum drag deceleration vs aerodynamic area-to-mass ratio.

follows that

$$Q^* = \frac{U^2\,|\sin\Gamma|\,b}{2[B-h^*+h_0]}\exp\left\{\frac{-b}{1+b}\right\} \tag{9}$$

If Eq. (4) is solved explicitly for h^* using Eq. (8), it can be shown that

$$B-h^*+h_0 = B\left[\frac{\rho(h_0)B}{b}\,\frac{k_\infty}{|\sin\Gamma|}\right]^{-1/1+b} \tag{10}$$

It then follows that

$$Q^* = \left[\frac{U^2\,|\sin\Gamma|\,b}{2B}\right]\left[\frac{\rho(h_0)B}{be^b}\,\frac{k_\infty}{|\sin\Gamma|}\right]^{1/1+b} \tag{11}$$

Since b is positive, it is clear that Q^* increases with increasing k_∞.

In the complete paper, similar results were obtained using the multilayer 1976 Standard Atmosphere model.[3] A different ρ vs h function was used in each layer, depending on its isothermal or adiabatic temperature characteristics. It was found that dependence of Q^* on k_∞ was governed by the function used to portray ρ vs h. Q^* was observed to be independent of k_∞ only in isothermal layers because an exponential density profile was used (see Fig. 1). In adiabatic layers, Q^* increased monotonically with increasing k_∞ because a power-law density profile was used.

References

[1] Allen, H. and Eggers, A., "A Study of the Motion and Aerodynamic Heating of Ballistic Missiles Entering the Earth's Atmosphere at High Supersonic Speeds," NACA TN 1381, 1958.

[2] Barbera, F., "Closed-Form Solution for Ballistic Vehicle Motion," *Journal of Spacecraft and Rockets,* Vol. 18, Jan.-Feb. 1981, p. 52.

[3] *U.S. Standard Atmosphere 1976,* U.S. Government Printing Office, Washington, D.C., 1976, p. 6.